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Calculation of the Energy of Elastic Deformation of Kevlar Backing Plates for Ceramic Armours

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Abstract

An equation is developed which enables the calculation of the elastic deformation energy of a Kevlar plate used as a backing for ceramic armour, following projectile impact. Knowledge of the plate profile at maximum deflection is required. Plate profiles were obtained by photographically recording ballistic impacts. The energy of deformation of the composite backing represents 30 to 36% of the projectile's initial kinetic energy as calculated for two light weight ceramic armours.

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1. *Introduction*

Light weight ceramic composite armours typically comprise a ceramic front plate bonded to a ductile backing plate as shown in Figure 1. The purpose of the ceramic is to erode and break up an impacting projectile while the role of the backing plate is twofold: it adds stiffness to the ceramic to delay the onset of tensile fracture in the ceramic, and undergoes deformation to absorb some of the kinetic energy of the impacting projectile. The emphasis on either of these latter mechanisms will shift depending on the material chosen for the backing plate.

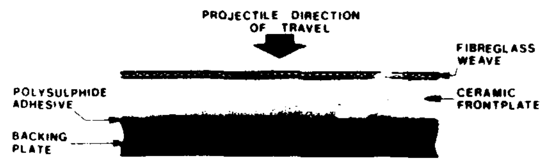

Figure 1: *Schematic representation of light weight ceramic armour.*

Earlier work [1] has identified the mechanisms of energy dissipation which operate during ballistic impact on an aluminium backed ceramic armour. The proportion of the projectile's initial kinetic energy dissipated through each of the identified mechanisms, including bending and stretching of the aluminium back-plate, was also determined [1]. The aim of the present work is to determine the amount of energy which is dissipated through bending and stretching of Kevlar plates when used to back ceramic armour. Kevlar was chosen as it is often used in light weight armours because it combines light weight with high stiffness and good ballistic resistance.

2. *Experimental*

Impact tests were conducted on ceramic armour targets comprising alumina (97.5% Al_2O_3) front plates and Kevlar backing plates which in turn consisted of a plain woven 223 g/m^2 Kevlar 49 fabric bonded with approximately 20% by weight of an ethylene vinyl acetate thermoplastic copolymer. Table 1 lists details of target component thicknesses and areal densities. Conical nosed 0.3 calibre projectiles of hardened steel weighing 5.6 grams and launched with the aid of a sabot at velocities up to 800 m/s from a 1/2 inch gas gun (designed at the US Army Materials Technology Laboratory [2]) were used for these tests.

A computer controlled Cranz-Schardin spark camera with a maximum frame speed of 10^6/s was used to photographically record impact events.

Table 1: *Thickness and areal densities of ceramic armour components*

Target	Alumina front plate		Kevlar back plate		Total Areal Density (kg/m^2)
	Thickness (mm)	Areal Density (kg/m^2)	Thickness (mm)	Areal Density (kg/m^2)	
2C	2	7.9	2.7	3.6	11.5
3D	3	11.2	4.1	5.4	16.6

3. *Experimental Results*

Several tests were conducted for each armour configuration in order to determine the velocity at which the projectile was just able to penetrate each armour configuration. Figures 2 and 3 are shadowgraphs of targets 2C and 3D respectively during impact recorded with the Cranz-Schardin camera. These figures show a side view of the targets just prior to perforation at near limit velocity (the velocity at which the projectile is just able to penetrate the armour) The contour of the rear surface of the backplate is evident in each figure. The considerable streaking evident in these figures is caused by light emitted from fragments as they are ejected from the impact site.

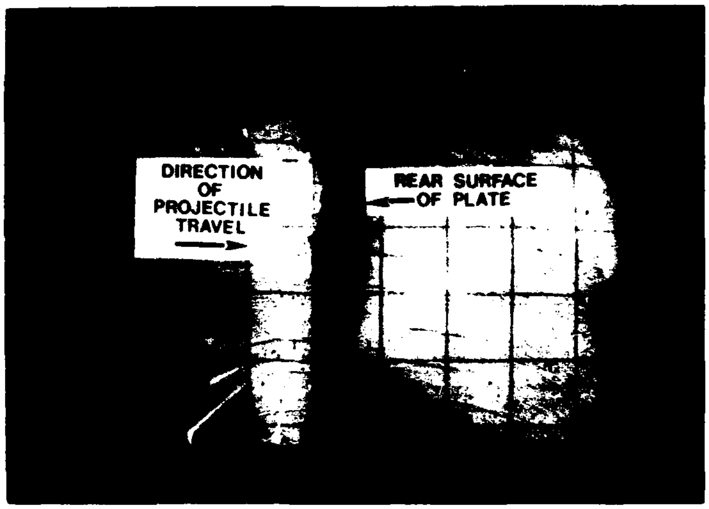

Figure 2: *Shadowgraph showing a side view of target 2C just prior to perforation by a projectile impacting from the left. The grid spacings are 1" square.*

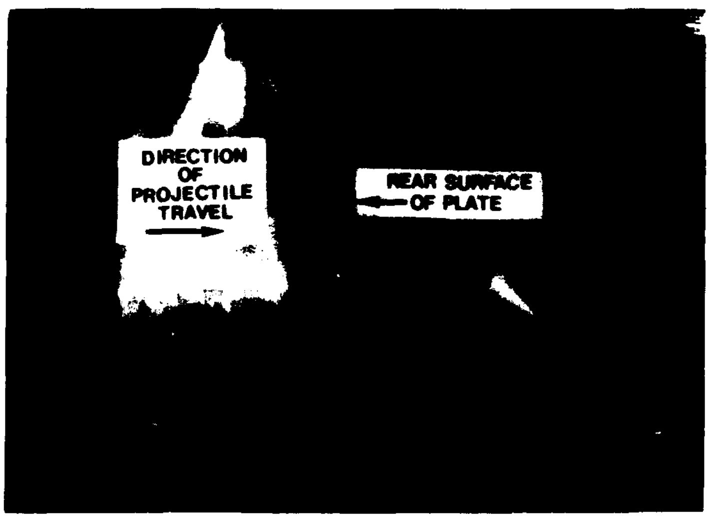

Figure 3: *Shadowgraph showing a side view of target 3D just prior to perforation by a projectile impacting from the left. The grid spacings are 1" square.*

The grid lines on Figures 2 and 3 have a one inch spacing and enable the rear profiles of these two Kevlar backing plates to be compared in Figure 4. Following impact the backing plates return almost to their original planar geometry although considerable amounts of fibre shearing, fibre pullout and delamination accompanies perforation [3].

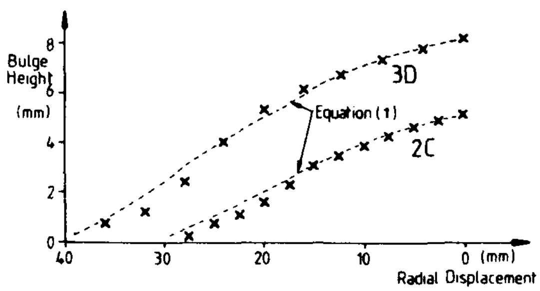

Figure 4: Graphical representation of the rear surfaces of the kevlar plates shown in Figures 2 and 3. A "best fit" of equation (1) to this data is also shown.

The equation for the deflection, w, of an isotropic circular elastic membrane loaded at the centre and clamped at the edges is given by Timoshenko [4] as

$$w = \frac{p}{16\pi D}\,(\alpha^2 - r^2) + 2r^2 \log(r/\alpha) \qquad (1)$$

where p is the load at the centre, D is the flexural rigidity of the membrane, α is the membrane radius and r is the radial coordinate varying from 0 at the membrane centre to α at the membrane boundary. The surface predicted by equation (1) is plotted in Figure 4 and represents a good approximation to the experimental deflections replotted from Figures 2 and 3. This, along with the return to a planar geometry at large times, suggests that the Kevlar backing plates behave in an approximately elastic manner during impact.

The velocity of the impacting projectile was determined by electronically recording the duration between interrupts of two laser beams directed across the projectile path and having a known separation. The kinetic energies of the

respective projectiles just prior to impact was 0.9 kJ for target 2C and 3.6 kJ for target 3D.

4. Theoretical Predictions

Here we will consider a circular elastic membrane loaded at the centre and clamped at its edge. The expression for the amount of energy required to cause the deflections shown in Figure 4 will be developed. Such an expression can be determined from consideration of either the work done by the stresses imposed on the plate or through the strain energy associated with the action of the bending moments on the plate. Whilst the latter approach will be followed here, the interested reader is referred to Appendix 1 where an identical equation is derived from the former approach.

Figure 5a shows a cross-section in the xz plane through a bent plate. If the plate surface in the z direction is described by $w(x, y)$ then the curvature of the surface in the xz and yz planes can be represented by $-\partial^2 w/\partial x^2$ and $-\partial^2 w/\partial y^2$ respectively. The work done by the uniformly distributed moments M_x and M_y acting within the xz and yz planes respectively on an element of the plate, $dxdydz$ (as shown in Fig. 5b), during bending is one half the product of the moment and the angle through which the surface is deflected. The angle of deflection caused by the moment $M_x dy$ is therefore

$$\left(\frac{\partial^2 w}{\partial x^2}\right)dx$$

and the work done within this plate element is

$$dW_x = \frac{-1}{2} M_x \left(\frac{\partial^2 w}{\partial x^2}\right) dx\,dy$$

and similarly

$$dW_y = \frac{-1}{2} M_y \left(\frac{\partial^2 w}{\partial y^2}\right) dy\,dx$$

and for the off axis moments

$$dW_{xy} = dW_{yx} = \frac{1}{2} M_{xy} \left(\frac{\partial^2 w}{\partial x\,\partial y}\right) dx\,dy$$

so that

$$dW = \frac{-1}{2}\left(M_x \frac{\partial^2 w}{\partial x^2} + M_y \frac{\partial^2 w}{\partial y^2} + 2M_{xy}\frac{\partial^2 w}{\partial x\,\partial y}\right)dx\,dy \qquad (2)$$

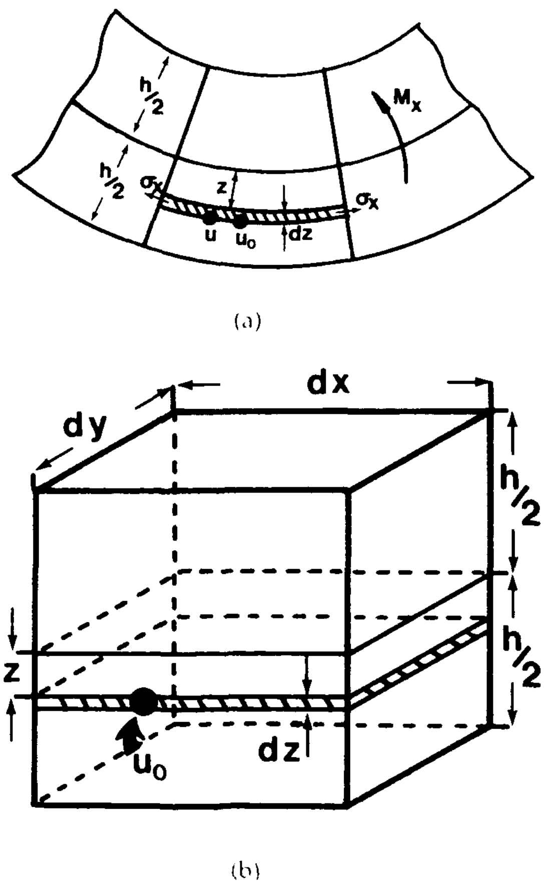

Figure 5: *Schematic representation of (a) a cross-section in the xz plane through a bent plate and (b) an element, dxdydz, within an undeformed plate of thickness h.*

The moments M_x and M_y are given by summing over all of our infinitesimal elements (see Fig. 5b) the respective stresses times the area over which they act multiplied by the moment arm with respect to the mid plane [5], i.e.

$$
\begin{aligned}
M_x &= \int_{-h/2}^{h/2} \sigma_x \, z \, dz \\
M_y &= \int_{-h/2}^{h/2} \sigma_y \, z \, dz \\
M_{xy} &= -\int_{-h/2}^{h/2} \tau_{xy} \, z \, dz
\end{aligned}
\tag{3}
$$

where σ_x and σ_y are the stresses in the x and y directions respectively and τ_{xy} is the shear stress in the xy plane. For a homogeneous orthotropic material in plane stress Hooke's law predicts the following stress strain relations

$$
\begin{aligned}
\sigma_x &= Q_x \epsilon_x + Q_{xy} \epsilon_y \\
\sigma_y &= Q_{yx} \epsilon_x + Q_y \epsilon_y \\
\tau_{xy} &= G_{xy} \gamma_{xy}
\end{aligned}
\qquad (4)
$$

where

$$
\begin{aligned}
Q_x &= E_x / (1 - \nu_{xy} \nu_{yx}) \\
Q_y &= E_y / (1 - \nu_{xy} \nu_{yx}) \\
Q_{xy} &= \nu_{yx} E_x / (1 - \nu_{xy} \nu_{yx}) \\
Q_{yx} &= \nu_{xy} E_y / (1 - \nu_{xy} \nu_{yx})
\end{aligned}
$$

Here E_x and E_y are the elastic modulus in the x and y directions respectively, G_{xy} is the shear modulus in the xy plane and ν_{xy} and ν_{yx} are the Poissons ratios in the same plane.

From Figure 5a we see that if point u_0 moves to location u during bending, we have $u = u_0 - z\,(\partial w/\partial x)$ and since the strain in the x direction is given by $\partial u/\partial x$ we have

$$\epsilon_x = -z \frac{\partial^2 w}{\partial x^2}$$

similarly

$$\epsilon_y = -z \frac{\partial^2 w}{\partial y^2} \qquad (5)$$

and

$$\gamma_{xy} = -2z \frac{\partial^2 w}{\partial x \partial y}$$

Substituting equations (4) and (5) into the equations for the bending moments (3) we get

$$M_x = \int_{-h/2}^{h/2} \left(-\frac{zE_x}{(1 - \nu_{xy} \nu_{yx})} \frac{\partial^2 w}{\partial x^2} - \frac{z \nu_{yx} E_x}{(1 - \nu_{xy} \nu_{yx})} \frac{\partial^2 w}{\partial y^2} \right) z\,dz$$

$$= \frac{-h^3 E_x}{12\,(1 - \nu_{xy} \nu_{yx})} \left(\frac{\partial^2 w}{\partial x^2} + \nu_{yx} \frac{\partial^2 w}{\partial y^2} \right)$$

similarly

$$M_y = \frac{-h^3 E_y}{12(1 - \nu_{xy}\nu_{yx})}\left(\frac{\partial^2 w}{\partial y^2} + \nu_{xy}\frac{\partial^2 w}{\partial x^2}\right) \tag{6}$$

and

$$M_{xy} = \frac{2h^3 G_{xy}}{12}\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)$$

We can now substitute equations (6) into (2) and integrate to obtain an expression for the total strain energy of bending. For a woven composite material of several layers we can assume a good degree of isotropy exists and we can make the approximation $E = E_x = E_y$ and $\nu = \nu_{xy} = \nu_{yx}$. Applying this approximation and introducing $D = Eh^3 / (1 - \nu^2)$ as the flexural rigidity of the membrane we have

$$W = \frac{D}{2}\int_0^{\alpha}\int_0^{\alpha}\left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1 - \nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2\right]\right\} dx\,dy \tag{7}$$

where we have utilized the relationship $G = E / (2(1 - \nu))$. For a membrane that is clamped around its edge the second term in the square brackets in equation (7) vanishes and we are left with

$$W = \frac{D}{2}\int_0^{\alpha}\int_0^{\alpha}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 dx\,dy$$

or in polar coordinates [4]

$$W = \frac{D}{2}\int_0^{2\pi}\int_0^{\alpha}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right)^2 r\,dr\,d\theta \tag{8}$$

In the previous section we determined that the deflection of the Kevlar back plate can be approximated by equation ( ). From this equation we see that maximum plate deflection, w_0, occurs for $r = 0$ and we have

$$w_0 = \frac{p\alpha^2}{16\pi D}$$

Rewriting equation (1) we have

$$w = \frac{w_0}{\alpha^2}\left[(\alpha^2 - r^2) + 4.6r^2 \ln(r/\alpha)\right]$$

Differentiating and substituting into equation (8) we get

$$W = \frac{D}{2} \int_0^{2\pi} \int_0^{\alpha} \left[\frac{w_0}{\alpha^2} \left(14.4 + 18.4 \ln (r/\alpha) \right) \right]^2 r \, dr \, d\theta$$

Completing the square and integrating by parts gives

$$W = \frac{\pi D w_0^2}{\alpha^4} \left[338.6 \frac{r^2}{2} \left(\ln^2 (r/\alpha) - \ln (r/\alpha) + \frac{1}{2} \right) + 530 \frac{r^2}{2} \left(\ln (r/\alpha) - \frac{1}{2} \right) + 207.4 \frac{r^2}{2} \right]_0^{\alpha}$$

$$= \frac{55.9 \pi D w_0^2}{\alpha^2} \tag{9}$$

5. *Calculation of Energy of Deformation*

The maximum deflection, w_0, for both targets 2C and 3D can be determined from Figure 4 and are approximately 5 mm and 8 mm respectively and the radius of plate deformation, α, 30 mm and 40 mm respectively.

The flexural rigidity, D, is dependent on both plate modulus and thickness and assuming $E_{Kevlar} = 31$ GPa and $\nu = 0.30$ we have, for target 2C with $h = 2.7$ mm

$$D = 55.0 \text{ J}$$

and for target 3D with $h = 4.1$ mm

$$D = 184.3 \text{ J}$$

The total energy of deformation is then given by equation (9) and is = 270 J for target 2C and $W = 1.29$ kJ for target 3D. These values represent 30% and 36% respectively of the initial kinetic energy of the impacting projectiles. While these values also account for the potential energy in the backing they do not include the energy dissipated through shearing, delamination, or other local deformation mechanisms. Nevertheless, these values for the strain energy compare well with the 20 to 50% calculated for the bending and stretching of an aluminium back plate from previous work [1].

6. *Conclusions*

The total strain energy of the Kevlar backing plate for a light weight ceramic armour deformed through ballistic impact can be approximated from knowledge of the profile of the backplate just prior to perforation and assuming

the elastic properties in the plane of the plate are approximately isotropic. The amount of energy dissipated through elastic deformation of a Kevlar backing plate when impacted at near limit velocity is 0.27 kJ and 1.29 kJ for ceramic armours comprising a 2 mm thick alumina front plate with 2.7 mm Kevlar back plate and a 3 mm thick alumina front plate with 4.1 mm Kevlar back plate respectively.

7. *Acknowledgements*

This work was performed while the author was attached to the US Army Materials Technology Laboratory in Boston, MA and owes much of its success to the capable assistance provided by Mr Don Stewart of AMTL.

8. *References*

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Appendix 1

Work done in an elastic membrane by the stress components is given by

$$W = \frac{1}{2}\int_V (\sigma_x \epsilon_x + \sigma_y \epsilon_y + \tau_{xy}\gamma_{xy})\, dV$$

where the integration is over the membrane volume V. From equation (4) we have

$$W = \frac{1}{2}\int_V \left[\frac{E_x \epsilon_x^2}{(1-\nu_{xy}\nu_{yx})} + \frac{\nu_{xy}E_x \epsilon_x \epsilon_y}{(1-\nu_{xy}\nu_{yx})} + \frac{\nu_{yx}E_y \epsilon_y \epsilon_x}{(1-\nu_{xy}\nu_{yx})} + \frac{E_y \epsilon_y^2}{(1-\nu_{xy}\nu_{yx})} + G_{xy}\gamma_{xy} \right] dV$$

substituting for the strains in (5) we have

$$W = \frac{1}{2}\,\frac{1}{(1-\nu_{xy}\nu_{yx})}\int_V \left[E_x\left(\frac{\partial^2 w}{\partial x^2}\right)^2 z^2 + \nu_{xy}E_x\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right) z^2 \right.$$
$$\left. + \nu_{yx}E_y\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right) z^2 + E_y\left(\frac{\partial^2 w}{\partial y^2}\right)^2 z^2 + 4G_{xy}(1-\nu_{xy}\nu_{yx}) z^2 \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \right] dV$$

If we assume $E = E_x = E_y$, $\nu = \nu_{xy} = \nu_{yx}$ and $dV = dxdydz$ then

$$W = \frac{E}{2(1-\nu^2)}\int_0^a\int_0^a\int_{-h/2}^{h/2} \left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2\nu\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right) + \frac{4G}{E}(1-\nu^2)\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \right] z^2\, dxdydz$$

$$= \frac{Eh^3}{24(1-\nu^2)}\int_0^a\int_0^a \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\nu)\left[\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right) - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \right] \right\} dxdy$$

which is identical to equation (7) determined from the action of bending moments.

List of Symbols Used

w - deflection of membrane surface

p - load at membrane centre

α - radius of the deflected membrane

r - radial coordinate from membrane centre to α

D - flexural rigidity

M - bending moment

E - elastic modulus

G - shear modulus

ν - Poissons ratio

σ - principle stress acting within the membrane

τ - shear stress acting within the membrane

ε - principal strain imposed within the membrane

W - work done within the membrane

h - membrane thickness

V - membrane volume

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